

Quantum Computing – Experimental Successes

ECE497NC lecture 24, April 21st, 2004

Lecture by Professor Nicholas P. Carter

Summary and Elaboration by Tuan Bui

Papers:

- P1. Toward Quantum Computation: A Five-Qbit Quantum Processor by Matthias Steffen, Lieven M.K. Vandersypen, and Isaac L. Chuang
- P2. Experimental Realization of an Order-Finding Algorithm with an NMR Quantum Computer by L.M.K. Vandersypen and I.L. Chuang. Appeared in Physics Review Letters, 2000.
- P3. Experimental Realization of Shor's Quantum Factoring Algorithm Using Nuclear Magnetic Resonance by L.M.K. Vandersypen, M. Steffen, G. Breyta, C.S. Yannoni, M. Sherwood, and I.L. Chuang. Appeared in Nature, 414, 2001

Lecture Overview:

Quantum computers are in their infancy. Experimental systems today use Nuclear Magnetic Resonance (NMR) to align the spins of atomic nuclei into quantum states. Grover's order-finding algorithm [B1] and Shor's factoring algorithm [B2] are examples of two programs that have been successfully implemented on experimental NMR quantum computers. The focus of the lecture was on NMR; however, other implementations will be briefly examined.

Introduction:

At this stage in quantum computational research, actual quantum computers are experimental in nature and are few and far between. There are certain requirements for a functional quantum computer:

- A system of qubits.
- Qubits are individually addressable and must be able to interact with each other.
- Qubits are able to be initialized to a known state.
- A computational result must be able to be extracted from the qubits by some measurement.

When a quantum system interacts with the environment, the system loses "coherence," which means that qubits lose their superposition of quantum states. This loss of coherence is caused by both external interference and intentional observation, and

superpositioned states cannot be restored – thus, experimental results must be achieved before decoherence happens so that they may be observed. This brings to the forefront a fifth requirement of quantum computers – the coherence time must be sufficiently long (i.e. long compared to an average quantum logic gate’s duration) so that many quantum operations may be performed before decoherence. Collectively, these five requirements are known as the DiVincenzo criteria as outlined by David DiVincenzo in 1995[B3].

Quantum Computing Implementations:

Manipulation of Trapped Ions:

The first attempts to create quantum computers used sub-microscopic assemblies of quantum spins, which were difficult to manipulate at a quantum level. A following experiment involved a single-ion system, which consisted of a single trapped beryllium molecule containing two qubits [B6]. This system involved either a Penning trap or a Paul trap. A Penning trap uses a combination of static electric and magnetic fields to confine a single atom, whereas a Paul trap (or RF trap) uses ponderomotive fields generated by inhomogeneous oscillating electric fields. The net effect is to hold an atom in place without unduly interfering with it.

To ensure consistent starting conditions, the trapped molecule is cooled to the “zero-point” state by resolved-sideband laser emitting polarized light. If the trapped ion is bombarded with photons of certain energy, it will emit photons of a lower energy, thus reducing the atom’s kinetic energy until the mean vibrational quantum number in the harmonic well reaches a small enough value.

By applying uniform electric fields (and thereby inducing a resonant force), or alternatively using a “moving standing wave” of laser radiation, experimenters were able to induce coherent and Schrödinger-cat states of motion in the trapped molecule. These states are significant because they represent superpositions of multiple states. Also, experimenters were able to construct a CNOT logic gate using two ground-hyperfine states of the trapped beryllium ion. A CNOT operation flips the second bit if and only if the first bit is asserted.

This methodology was limited because at the time, only about ten operations could be performed before decoherence. Also, only one ion could be manipulated at a time, severely restraining the usefulness of the trapped ion technique. However, recent strides in trapped ion technology have produced four trapped and entangled ${}^9\text{Be}^+$ ions [B8], and also realized the Deutsch-Jozsa algorithm using a single trapped calcium-40 ion [B9].

Nuclear Magnetic Resonance (NMR):

The advent of using nuclear magnetic resonance to control weakly polarized macroscopic ensembles of spins brought realizable quantum computing out of the theoretical and into the realizable, as it opened up many-qubit systems that were comparatively easy to manipulate [B4].

NMR as applies to quantum computing relies on the existence of spin-1/2 nuclei. These nuclei have their own magnetic fields, and this field’s relation to an external magnetic field determines its quantum state. If the field is perfectly aligned or perfectly

out of alignment with the external field, then that is considered quantum 1 or 0 (spin up or spin down). The fact that this field is not restricted to just these two states, and can exist in any configuration between 0 and 1, allows it to be in a quantum mechanical superposition of multiple states. Thus, the spin of a nucleus serves as a quantum bit.

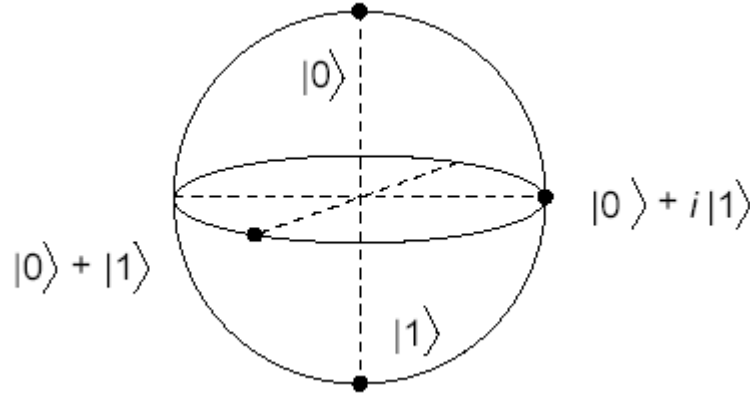


Figure 1: Pictorial abstraction of four different qubit states [P1].

Nuclear spins have long coherence times – often several seconds in length, and sometimes thousands of seconds [B5]. This easily trumps the several-nanosecond coherence times of electron spins in solids.

The current favored approach to combining several of these nuclei into one cohesive, easily-manipulated system is by using a custom molecule. This is the easiest approach – burying the atoms in a bulk material or else spreading them evenly across a surface is currently beyond our technical means.

Nuclear magnetic resonance has been around since the 1950’s, and has been used for imaging applications. Previous non-quantum applications involved complex sequences of arbitrarily-shaped RF pulses – a perfect match for application to quantum manipulation of nuclear spins.

By applying magnetic fields in the plane perpendicular to the static magnetic field, single-bit operations can be performed. When a spin-1/2 nucleus is placed in a static magnetic field, the spin of the nucleus will precess about the field’s axis. This is analogous to a top precessing about the gravity axis. The frequency of this precession falls in the RF range.

To manipulate this spin, we can apply a transverse RF field with a frequency resonant with the spin precession frequency. This will rotate the spin axis about the transverse plane – and this axis is determined by the phase of the RF field applied. The rotation angle is proportional to the duration and amplitude of the RF field.

With these tools, it is possible then to perform arbitrary one-bit operations. For example, a NOT operation could be performed on a qubit, flipping the state of a spin from up to down or down to up, with a carefully-orchestrated RF pulse. An RF pulse with half the duration would rotate a spin from up ($|0\rangle$) to a superposition of up and down ($1/\sqrt{2} |0\rangle + 1/\sqrt{2} |1\rangle$).

This works fine if we are manipulating a single nucleus in isolation. To perform operations on a single nucleus in the presence of other spin-1/2 nuclei, each nucleus must be individually addressable – and, in the case of special molecules, they are. This is

because different kinds of atoms have different resonant frequencies ν , and the same kind of atom in a different position in the same molecule will also have varying resonant frequencies if the host molecule has enough asymmetry in its structure. This is called chemical shift.

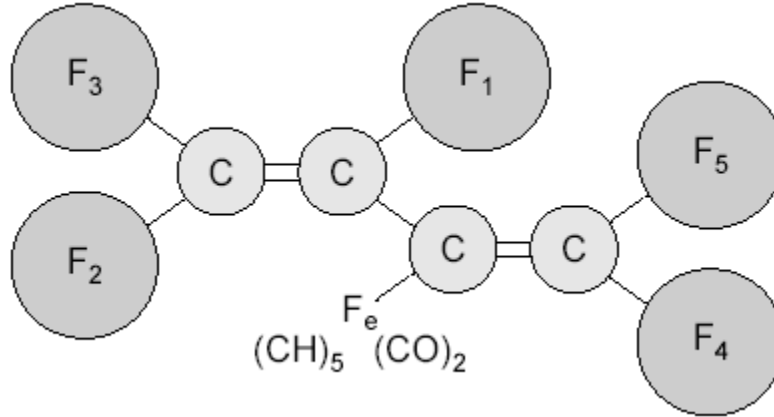


Figure 2: Pentafluorobutadienyl cyclopentadienylcarbonyliron complex with five fluorine-19 molecules containing spin-1/2 nuclei. Each spin is seperably addressable and is pairwise coupled with another spin [P1].

Figure 2 shows a molecule that was used in a 5-qubit quantum computer. The five ^{19}F atoms in this molecule exhibit enough chemical shift such that each is individually addressable. Furthermore, the chemical bonds between the ^{19}F atoms naturally link them together, so that an action on one spin-1/2 nucleus affects other spin-1/2 nuclei chemically linked to it.

	F5	F4	F3	F2
F1: 0	14.5	6.5	25	.114
F2: 17107	13	2	80	
F3: 22528	4	2.5		
F4: 37195	41			
F5: 47113				

Table 1: Relative chemical shifts of the ^{19}F spins at 11.7 T [Hz] and the J couplings [Hz]. A total of 76 out of the 80 lines in the 5 spectra are resolved [P2].

The degree of connectedness between pairs of spins is expressed by J , where $J/2$ is the difference in one spin's precession frequency when its pairmate switches between up ($|0\rangle$) and down ($|1\rangle$). J is expressed in frequency units. If the resonant frequency of a pair of spins is sufficiently different (i.e. many multiples of the pair's J value), then it is possible to manipulate each spin separately, yet still be able to track the actions of one spin by observing the other spin – thus, the two spins are considered coupled.

A crucial two-qubit operation in the quantum world is the CNOT gate. One way to implement this gate is as follows: if we apply a narrowband 180-degree RF pulse with frequency $\nu_1 + J_{12}/2$, where ν_1 is the resonant frequency of spin 1, and J_{12} is the coupling frequency between spin 1 and spin 2, then spin 2 is inverted if and only if spin 1 is $|1\rangle$. By using combinations of CNOTs, it is possible to swap the states of two qubits. In this

manner, operations between qubits that are not directly coupled to each other are still possible by swapping states until the desired state is coupled with the desired qubit.

At room temperature, the thermal equilibrium state of a spin is random; an up state or a down state is almost equally likely, with a skew of only 1 in 10^5 . To circumvent this problem, a procedure exists to utilize the structure inherent in thermal equilibrium [B5]. A perturbation is inserted into the system's large density matrix, which acts like a smaller-dimensional effective pure state. In this manner, a computation performed on a bulk of molecules that start at undetermined states can be interpolated such that a result is easily discernible. As a bonus, the impact of outside interference is further reduced, as a large body of molecules is less susceptible to noise than a single one. In effect, instead of operating on one molecule, a test tube with a number of molecules on the order of 10^{18} is used, each molecule acting as an independent quantum computer.

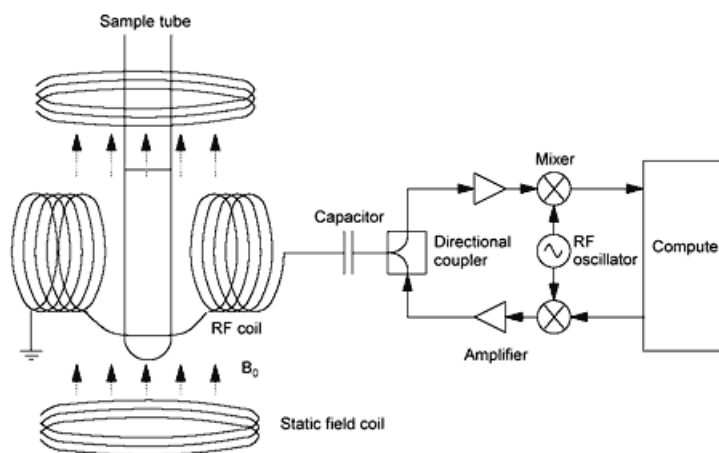


Figure 3: Diagram of an NMR spectrometer [P1].

In order to read the results of a quantum computation, a read-out pulse rotates spins whose state is $|0\rangle$ or $|1\rangle$ such that the spins now precess about the axis of the external magnetic field. Since they precess at a specific resonance frequency, they produce a signature time-variant magnetic field. This field is strong enough to be recorded. Also, because of the known information about the coupling of spins, it is possible to observe extra information from the frequency spectrum of just one spin.

A quantum computer, in this case, is no more than a single specialized molecule. The apparatus above in figure 3 is then the equivalent of a keyboard and monitor to interface with the 10^{18} quantum computers contained in the sample tube. These molecules are suspended in solvent. The static field coil is suitably strong (on the order of 10 tesla, or 200,000 times that of the earth's magnetic field) to produce spin resonance frequencies between 100 to 500 MHz. Higher field strength equates to higher chemical shifts. The Helmholtz RF coils produce the pulses necessary to manipulate the spin-1/2 nuclei. Usually, pulses are between $10\mu\text{s}$ and 10ms . The same coils are sensitive enough to record resultant fluctuations in magnetic fields in order to "read" results from the quantum computers in the sample tube.

NMR quantum computing has enjoyed a number of successful implementations of quantum algorithms, three of which are detailed later in this article.

Quantum Computing Algorithms:

Deutsch's algorithm was the first quantum algorithm to be described, and also the first to be implemented [B7]. When an unknown function f takes in one bit and outputs one bit, there can only be four principles by which it operates. Deutsch's algorithm manages to figure out which function the unknown is by using only one evaluation if the parity of function f is known. This previously could not be accomplished, since it is impossible in classical computing. This algorithm was successfully implemented using two spins on first a cytosine molecule, and then a ^{13}C -labeled chloroform molecule using NMR quantum computing.

Grover's quantum search algorithm, on the other hand, searches a large number of inputs for one specific set that produces a positive function output [B1]. If there are n bits possible in the inputs, then there are 2^n possible input strings – and only k of these input strings produces $f = 1$. In classical computing, this is an order $O(2^n)$ algorithm, taking $2^n/k$ trials on average, but Grover's algorithm takes $O(\sqrt{2^n}/\sqrt{k})$ in quantum format. The case of $n = 2$ was successfully demonstrated using quantum computing.

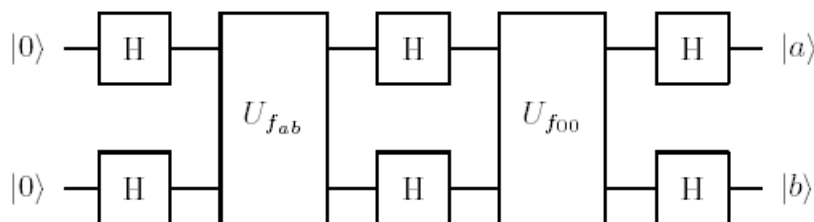


Figure 4: A quantum circuit for the implementation of Grover's quantum search algorithm on a two qubit computer. Boxes marked U are Hadamard gates. The first two qubit gate $U_{f_{ab}}$ corresponds to evaluation of the function f_{ab} , replacing an eigenstate $|ij\rangle$ by $-|ij\rangle$ if $i = a$ and $j = b$, while $U_{f_{00}}$ simply replaces $|00\rangle$ by $-|00\rangle$ [B7].

Shor's algorithm uses two major components working together to find the order of a permutation. Loosely explained, it is used when we want to find the order R of an algorithm, where R is the number of times we have to iterate the algorithm to return to our original starting value. In order to solve this problem using quantum computing for $4! = 24$ permutations on 4 input elements, a five-qubit molecule was synthesized.

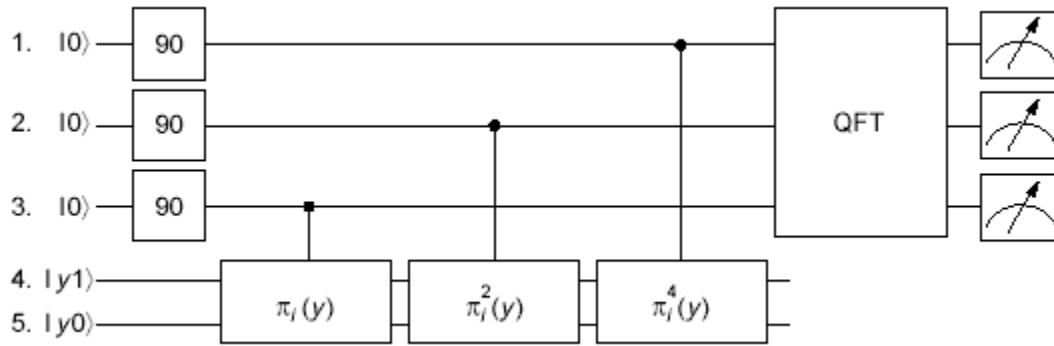


Figure 5: Quantum circuit implementing the order-finding algorithm for a permutation π . Each horizontal line corresponds to one qubit, and the boxes represent operations acting on the qubits (time goes from left to right). The vertical lines connecting a black dot and a box denotes that the box is to be executed if and only if the qubit indicated by the black dot is set to 1. Copyright © 2000, The American Physical Society [P1].

In a nutshell, a sequence of numbers repeating periodically will produce spikes in an FFT of that sequence. Using quantum parallelism, it is possible to evaluate the function over all possible inputs simultaneously. The resultant spikes in the FFT denote which inputs produce the desired solution.

Conclusions:

There are two predominant experimental methods for quantum computing today. One is trapped-ion computing, which works with small numbers of ions yet shows some scalability, and the other is NMR, which works with vast numbers of molecules in parallel. There is active development in both arenas, and the inroads gained in each technology are not entirely exclusive, either. Pulsed-RF techniques used in NMR have been successfully modified to work with trapped ions [B9]. Neither technique is yet at a point where commercial applications are viable.

By 9 AM on the day this article is being finalized, thirteen articles on quantum physics have been submitted to the arXiv e-print archives, four of which have to do directly with quantum computing. As one of the hot topics of physics and computational research, quantum physics is enjoying a surge of research interest as it moves out of its nascence. Considering the magnitude of investigation going on in the quantum computing arena, progress towards practical quantum computing is undoubtedly being made quickly and assuredly.

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